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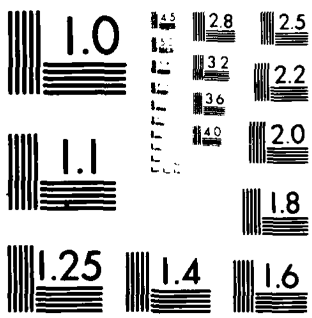

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Principles and Applications of Radio Frequency Impedance Probes

S. T. LAI

27 July 1981

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SPACE PHYSICS DIVISION PROJECT 7661
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AIR FORCE SYSTEMS COMMAND, USAF

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FOR THE COMMANDER

John Howard

Chief Scientist

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Dielectric function	Resonance	Ionosphere
Sheath	Hybrid frequency	Probe

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The underlying principle of the radio frequency plasma probe is presented. The resonance properties of the probe's equivalent circuit with and without external magnetic field are analyzed. The admittance of the equivalent circuit is shown to possess a pole at the sheath hybrid frequency and a zero at the upper hybrid frequency. The phase of the admittance changes signature to become negative, in the range of frequency between the pole and the zero. Some significant experiments using radio frequency plasma probe for ionospheric research are briefly described.

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Preface

The author wishes to thank H. A. Cohen for suggesting this work, stimulating discussion, and kind appreciation. He also thanks B. N. Maehlum for an interesting discussion on the use of RF probes.

Contents

Illustrations

Principles and Applications of Radio Frequency Impedance Probes

I. HISTORICAL INTRODUCTION

In 1957 Yeung and Sayers[1] reported that the RF impedance between two electrodes immersed in a plasma showed a minimum at a certain frequency ω_R (Figure 1). They believed it was the plasma frequency ω_p. In 1960 Takayama et al[2] and Miyazaki et al[3] examined a Langmuir probe response with an RF modulation on its dc voltage and found a sharp increase of current at ω_R (Figure 2). They, too, believed it was plasma frequency ω_p.

In 1963 Levitskii and Shashurin[4] used two fine wires as transmitter and receiver of RF signals across a plasma medium and discovered that the resonance

(Received for publication 22 July 1981)

1. Yeung, T.H.Y. and Sayers, J. (1957) An RF probe technique for the measurement of plasma electron concentrations in the presence of negative ions, Proc. Phys. Soc. 70B:663-668.
2. Takayana, K., Ikegami, H. and Miyazaki, S. (1960) Plasma resonance in a radio-frequency probe, Phys. Rev. Lett. 5:238-240.
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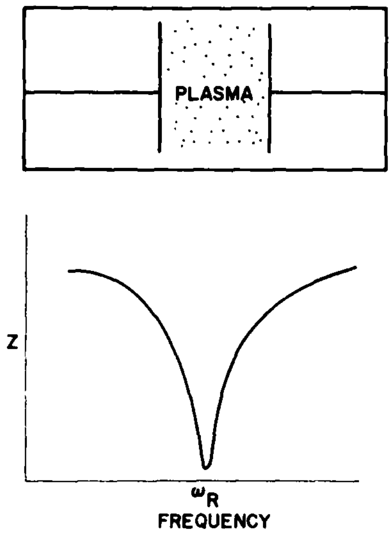

Figure 1. Yeung and Sayers' Experiment: Minimum Impedance at Frequency ω_R

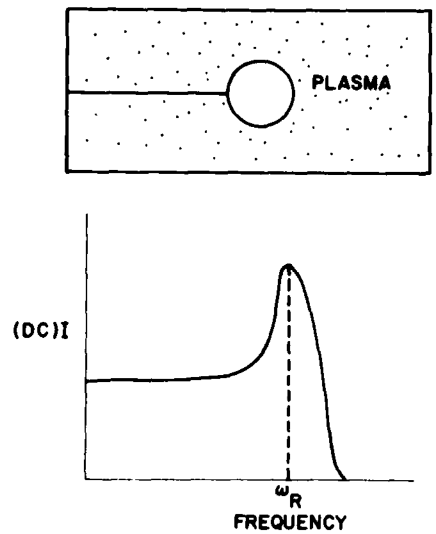

Figure 2. RF Modulation on the dc Voltage of a Langmuir Probe

frequency depended on sheath thickness. In 1964 Harp[5] showed that the frequencies of these experiments should coincide, depending on sheath thickness, and be slightly below the plasma frequency.

2. UNDERLYING PRINCIPLE

In a plasma, without magnetic fields and assuming ions being relatively immobile compared with electrons, the dielectric function $\varepsilon(\omega)$ is given by

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2} . \qquad (1)$$

When the applied frequency ω becomes equal to ω_p, the value of $\varepsilon(\omega)$ becomes zero, a resonance situation.

5. Harp, R.S. (1964) An analysis of the behavior of the resonance probe, Appl. Phys. Lett. 4:186.

Consider now a simple parallel plate probe system, as in Figures 3 and 4. Theoretical work of Harp (1964)[5] and Crawford and Harp (1965)[6] suggested that the simple plasma probe system can be studied as an equivalent circuit as in Figure 4. Ignoring L_p for smallness, we have a series of C_s - C_p - C_s.

$$\frac{1}{C_{eff}} = \frac{1}{C_s} + \frac{1}{C_p} + \frac{1}{C_s} \tag{2}$$

where,

$$C_s = \varepsilon_s \frac{A}{S} \approx \frac{A}{S} ,$$

where A is a cross sectional area of probe, and ε_s($\approx$ 1) is the dielectric constant of vacuum sheath.

$$C_p = \varepsilon_p \frac{A}{p} = \left(1 - \frac{\omega_p^2}{\omega^2}\right) \frac{A}{p}$$

$$\frac{1}{C_{eff}} = \frac{d}{\varepsilon_s A} \left[\frac{\omega^2 - \omega_p^2/(1 + p/2S)}{\omega^2 - \omega_p^2}\right]$$

so that,

$$C_{eff}(\omega) = C_o \left[\frac{\omega^2 - \omega_p^2}{\omega^2 - \omega_p^2/(1 + p/2S)}\right] \tag{3}$$

where,

$$C_o = \varepsilon_s \frac{A}{d} ,$$

and

$$d = p + 2S$$

Since current I(ω) is given by $I(\omega) = V(\omega)/Z(\omega)$,

$$I(\omega) = j\omega\ V(\omega)\ C(\omega) . \tag{4}$$

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6. Crawford, F.W. and Harp, R.S. (1965) The resonance probe – A tool for ionospheric and space research, J. Geophys. Res. 70:587.

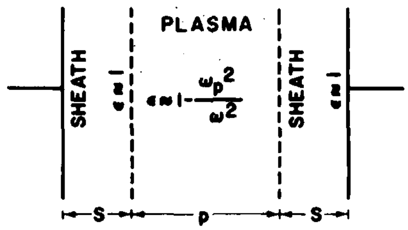

Figure 3. Probe System With Sheaths

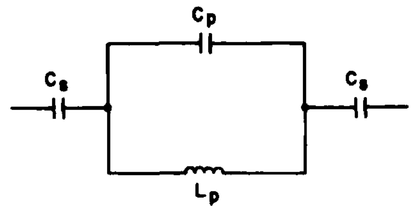

Figure 4. Equivalent Circuit

For the system considered,

$$I(\omega) = j\omega\, V(\omega)\, C_o \left[\frac{\omega^2 - \omega_p^2}{\omega^2 - \underbrace{\omega_p^2/(1 + p/2S)}_{\omega_R^2}} \right] \tag{5}$$

In Eq. (5) the current $I(\omega)$ or admittance $1/|Z(\omega)|$ peaks at $\omega = \omega_R$ and has a zero at $\omega = \omega_p$, (Figure 5). The peak is finite due to the presence of resistance.

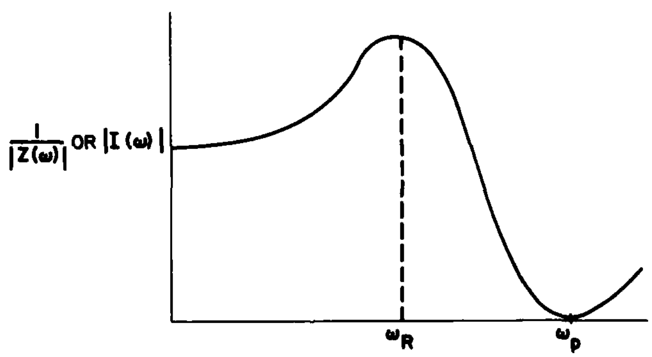

Figure 5. Resonances of Current in Plasma Condenser System

Harp (1964)[5] also showed that for a spherical RF probe, the resonance frequency ω_R is given by

$$\omega_R^2 = \frac{\omega_p^2}{1 + R/(k\,\lambda_D)} \tag{6}$$

where R is radius of probe, λ_D Debye radius, and k an empirical constant.

3. PLASMA WITH MAGNETIC FIELD

In the presence of magnetic field $\vec{B}$ the system becomes anisotropic. Charged particle motion parallel to $\vec{B}$ is unaffected by $\vec{B}$ because the Lorentz force $e(\vec{V} \times \vec{B})$ is zero. For $\vec{V}$ not parallel to $\vec{B}$, helical motion results (Figure 6). The dielectric function ϵ becomes a matrix, or so-called dielectric tensor, due to the anisotropy of space. The principal modes, parallel to or perpendicular to $\vec{B}$, are relatively simple

$$\epsilon_{\parallel} = 1 - \frac{\omega_p^2}{\omega^2}, \qquad \epsilon_{\perp} = 1 - \frac{\omega_p^2}{\omega^2 - \omega_c^2}, \tag{7}$$

where $\omega_c = \frac{eB}{m}$ is called cyclotron frequency (or gyrofrequency).

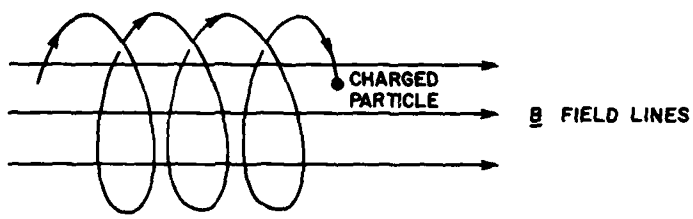

Figure 6. Helical Particle Trajectory

It is known that the frequency of plasma oscillations perpendicular to magnetic field is

$$\omega_{UHF} = \sqrt{\omega_p^2 + \omega_c^2}, \tag{8}$$

which is the frequency when $\varepsilon_{\perp}(\omega) = 0$. For some reason, this frequency is called the upper hybrid frequency. (The lower hybrid frequency is for ions.) The plasma frequency parallel to $\vec{B}$ field is unchanged because $\varepsilon_{\parallel}(\omega_p) = 0$.

Consider again the series $C_s - C_p - C_s$ capacitance, as in Figure 3. With $\vec{B}$ field perpendicular to probe electric field lines, we have again

$$\frac{1}{C_{eff}} = \frac{1}{C_s} + \frac{1}{C_p} + \frac{1}{C_s}$$

where,

$$C_s \approx \frac{A}{s}$$

and

$$C_p \approx \frac{A}{p} \frac{1}{\left(1 - \frac{\omega_p^2}{\omega^2 - \omega_c^2}\right)} .$$

The result is:

$$C_{eff} = C_o \left[\frac{\omega^2 - \omega_{UHF}^2}{\omega^2 - \underbrace{(\omega_c^2 + \frac{2S}{d} \omega_p^2)}_{\substack{\omega_{SHR}^2 \\ \text{Sheath Hybrid Resonance}}}}\right] \tag{9}$$

where,

$$C_o = \frac{A}{d}$$

$$d = p + 2S .$$

In the limit of vanishing magnetic field, we find

$$\lim_{B \to 0} C_{eff} = C_o \left[\frac{\omega^2 - \omega_p^2}{\omega^2 - (0 + \frac{2S}{d} \omega_p^2)} \right] = C_o \left[\frac{\omega^2 - \omega_p^2}{\omega^2 - \omega_p^2/(1 + p/2S)} \right],$$

which is the result obtained for no-magnetic field case in Eq. (3).

4. PHASE AND AMPLITUDE OF ADMITTANCE

The properties of the complex admittance, derived in the previous section, are now investigated in more detail. The admittance is (Figure 7)

$$\frac{1}{Z(\omega)} = j\omega C_o \left[\frac{\omega^2 - \omega_{UHF}^2}{\omega^2 - \underbrace{(\omega_c^2 + \frac{2S}{d} \omega_p^2)}_{\omega_{SHR}^2}} \right] \tag{10}$$

where,

$$C_o = \frac{A}{d}$$

$$d = p + 2S .$$

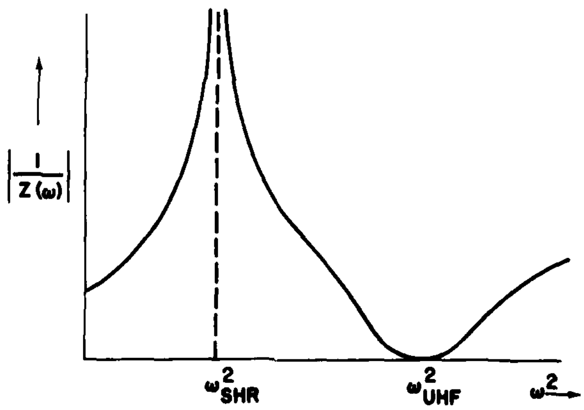

Figure 7. Poles and Zeros of Admittance of RF Probe

In reality there is always energy loss, due to particle collisions, heating of plasma medium, excitation of plasma waves, etc. In other words, there is a resistance in the RF probe. This fact renders the infinity of the amplitude of admittance to be finite.

In a complex plane (Figure 8) a vector $\vec{A}$ represented by its phase and amplitude, defined by

$$\text{Phase } \theta = \tan^{-1}\left(\frac{\text{Im } \vec{A}}{\text{Re } \vec{A}}\right) \tag{11}$$

and

$$\text{Amplitude } |\vec{A}| = \sqrt{(\text{Im } \vec{A})^2 + (\text{Re } \vec{A})^2} \quad . \tag{12}$$

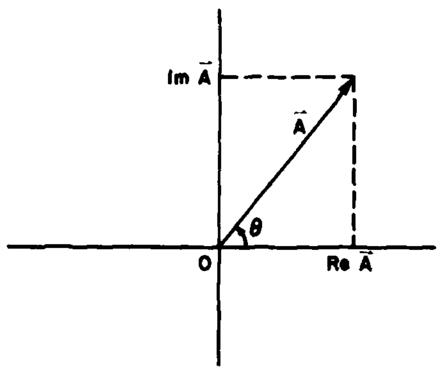

Figure 8. Amplitude and Phase of a Complex Vector

With resistance in the RF probe, the behavior of the admittance of the probe is shown in Figure 9. It is remarkable that as frequency increases beyond sheath hybrid resonance, the phase changes signature, indicating that the system behaves now as an inductance. It remains behaving inductively until the frequency increases beyond ω_{UHF}, the upper hybrid frequency. Then it becomes a capacitance again.

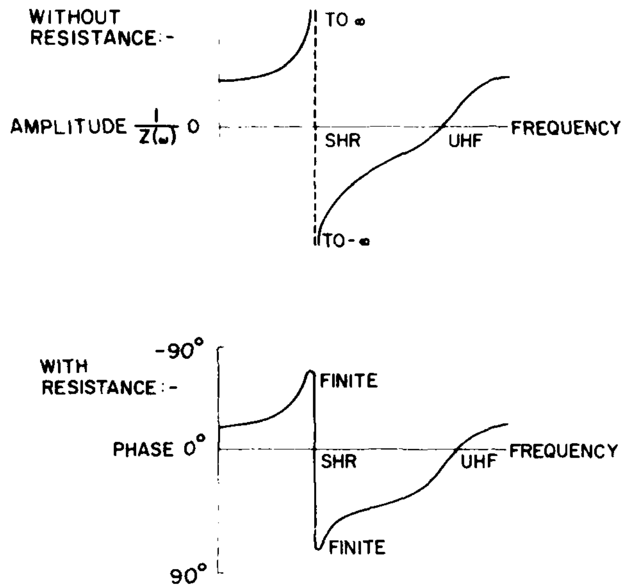

Figure 9. Phase and Amplitude of Admittance of RF Probe

5. TECHNIQUES OF USING RF PROBES FOR IONOSPHERIC RESEARCH

Some of the noted experiments using RF probes for ionospheric research are now reviewed. Mackenzie and Sayers used a parallel plate capacitor on Aeriel-3 satellite for measuring electron density at various altitudes.[7] Figure 10 shows a simplified diagram of their instrument circuits. A sawtoothed voltage sweep was applied on the capacitor. When the capacitance was minimum the effective plasma density was maximum in the capacitor. Thus, the sheath effect was minimized but not eliminated. The electron density was deduced from the plasma frequency obtained. Figure 11 shows the electron density deduced plotted versus altitude. The general behavior of the graph agrees with that obtained by other methods such as Langmuir probe method. The Langmuir probe method is known to be not too accurate. Note that the upleg measurements are generally higher than the downleg measurements for some unclear reasons, perhaps outgassing from the rocket.

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7. Mackenzie, E.C. and Sayers, J. (1966) The radio frequency electron density probe for rocket investigation of the ionosphere, Planet. Space Sci. 14:731.

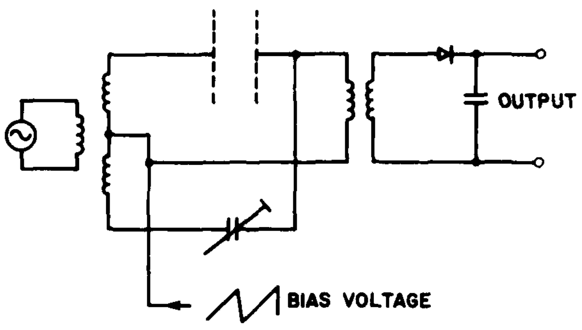

Figure 10. RF Probe Circuit Arrangement With dc Bias to Reduce Sheath Thickness [from Mackenzie and Sayers (1966)[7]]

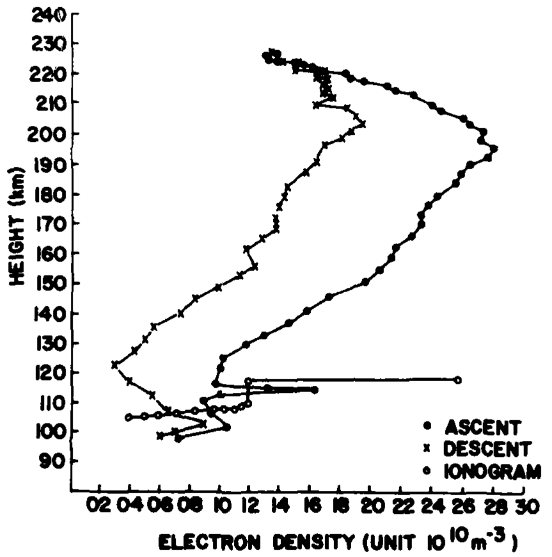

Figure 11. Electron-density Profile Obtained With the Birmingham University Probe [from Mackenzie and Sayers (1966)[7]]

In later experiments short cylindrical antennas or spheres were often used. Instead of applying sawtoothed voltage to minimize sheath effect, the upper hybrid frequency resonance was used. The sheath hybrid resonance depends on the shape of the capacitor and the profile of sheath density. Therefore it is not usable. Some of these experiments were performed by Melzner (1971)[8] on Nike-Apache rockets, Oya and Obayashi (1966,[9] 1967,[10] 1968[11]) using KAPPA 8, KAPPA 9, and L3H rockets and Oya and Morioka (1975)[12] on TAIYO Satellite. All of these experiments used the upper hybrid frequency resonance technique. An accuracy with ± 1 percent over the frequency range 0.7-17 MHz in 0.3 sec has been claimed [Ejiri and Obayashi (1970)[13]]. Examples of data records obtained in some of these experiments are shown in Figures 12 and 13. Probable ranges of electron and magnetic field parameters to be encountered in ionospheric and space application are shown in Figures 14 and 15.

Recently, radio frequency impedance probes have been used for the observation of electron beam-induced effects in a "mother-daughter" rocket configuration [Maehlum et al (1980)[14]].

6. DISCUSSION

The advantage of the upper hybrid frequency resonance technique is its independence of sheath effect. In Eq. (10) the sheath effect is in the denominator while the numerator is unaffected. It is the numerator that governs the zero of the admittance. It is true that the zero may be interfered by noise level, but the phase can provide an accurate zero crossing at the same frequency ω_{UHF}.

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8. Melzner (1971) Plasma Waves in Laboratory and Space, J. Thomas (ed.) **2**.
9. Oya, H. and Obayashi, T. (1966) Measurement of ionospheric electron density by a gyro-plasma probe: A rocket experiment by a new impedance probe, Rep. Iono. Space Res. Japan **20**:199.
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12. Oya, H. and Morioka, A. (1975) Instrument and observations of gyro-plasma probe installed on TAIYO for measurement of ionospheric plasma parameters and low energetic particle effects, J. Geomag. Geoelectr. **27**:331.
13. Ejiri, M. and Obayashi, T. (1970) Measurement of ionosphere by the gyro-plasma probe, J. Geomag. Geoele. 1-12.
14. Maehlum, B.N., Maseide, K., Aarsnes, K., Egeland, A., Grandal, B., Holtet, J., Jacobsen, T.A., Maynard, N.C., Soraas, F., Stadsnes, J., Thrane, E.V., Troim, J. (1980) Polar 5 – An electron accelerator experiment within an aurora. 1. Instrumentation and Geophysical Conditions, Planet. Space Sci. **28**:259-278.

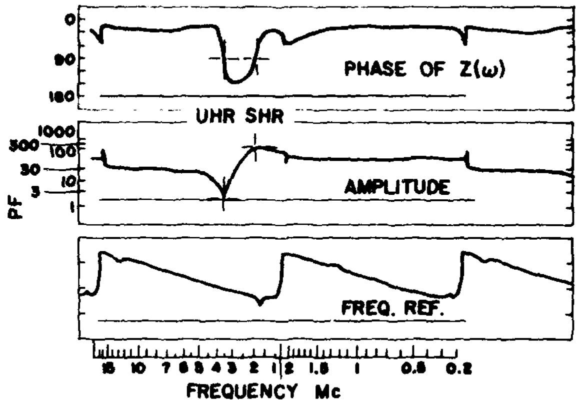

Figure 12. Example of the Observed Record of Resonance Effects at the Altitude of 570 km Showing the Upper Hybrid Resonance (UHR) Frequency (4.0 MHz) and Sheath Resonance (SHR) Frequency (2.0 MHz) [from Oya and Obayashi (1968)[11]]

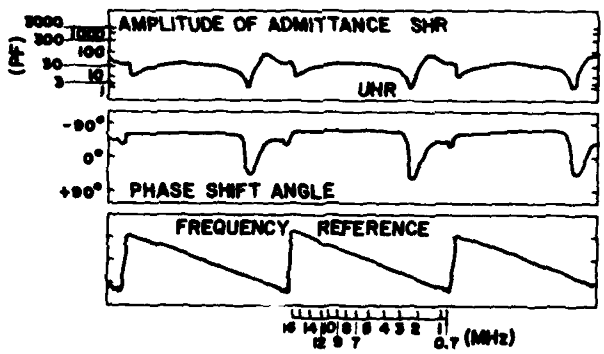

Figure 13. Example of the Record Obtained at the Altitude of 172 km (f_{UHR} = 2.6 MHz, f_{SHR} = 1.7 MHz) [from Ejiri and Obayashi (1970)[13]]

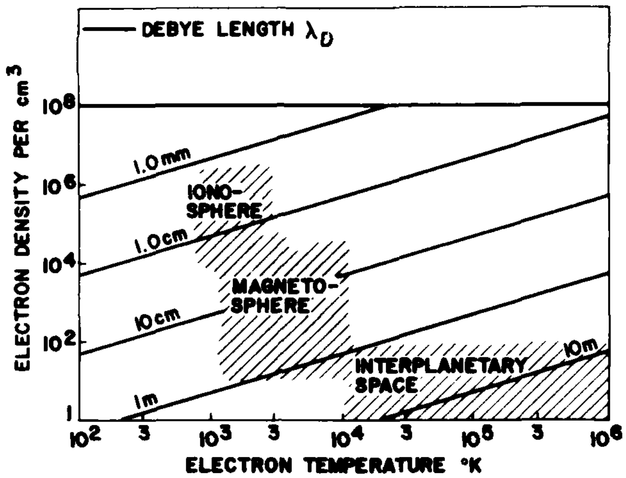

Figure 14. Probable Range of Electron Density and Temperature to be Encountered in Ionospheric and Space Applications [from Crawford and Harp (1965)[6]]

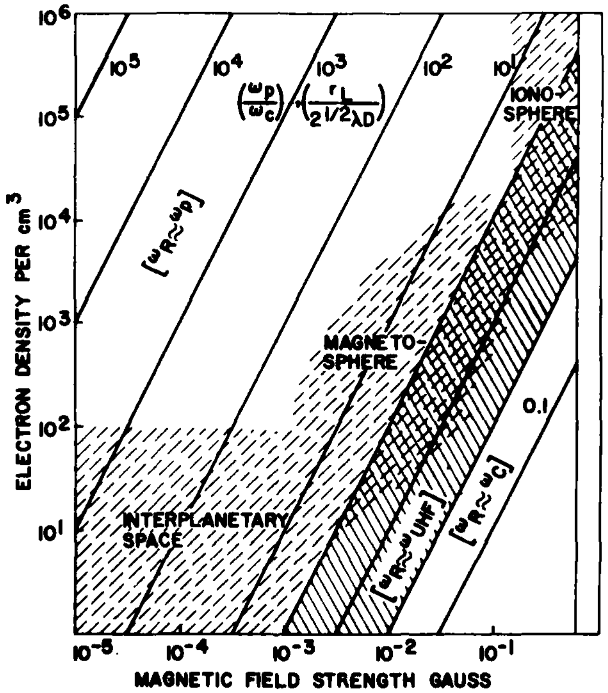

Figure 15. Ranges of Application of the Radio Frequency Probe. ω_R is the frequency at the zero of admittance, and r_L is the Larmer radius [from Crawford and Harp (1965)[6]]

Furthermore, it can be shown (Appendix) that the numerator of Eq. (10) is independent of the profile of sheath density, as long as the sheath distance is less than d/2. The cyclotron frequency ω_c can be measured accurately by using on-board magnetometers. Then plasma frequency is given by $\sqrt{\omega^2 - \omega_{UHF}^2}$.

Not only the frequency of the zero of admittance is unaffected by the density profile of the sheath around a probe, but also it is insensitive to the geometry of the sheath and probe.

There are certain limitations on the capacitance probe methods. The capacitance is proportional to A, the area of capacitor surface. If A is small, the measured value of capacitance would be small. This fact renders the use of capacitance probes to measure the plasma density in a small area practically impossible.

Another limitation is the time needed to sweep a frequency range. Since a rocket or space vehicle is traveling at high speed, a long time span would mean large uncertainty in position.

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Appendix A

Theorem: The Independence of the Zero of Admittance on the Sheath Electron Density Profile

For simplicity and clarity of discussion, a vacuum sheath has been used throughout in the text. It has been shown that the admittance of a capacitor, with a vacuum sheath, has a zero at ω_p (or ω_{UHF} if magnetic field is present), and a pole at ω_{SHR}, which is due to sheath resonance. While the pole depends on sheath property, the zero is unaffected. It is this remarkable property that renders the use of the radio frequency impedance probe an accurate method for determining plasma density. It would be interesting to conjecture that this remarkable property remains true for a general sheath electron density profile; it will be shown to be true in this appendix.

We generalize the probe system shown in Figure 3. We consider a sheath-plasma-sheath in series ($p > 0$, $s > 0$), but the sheaths are no longer vacuum. Near the probe walls, electron density decreases monotonically with the distance from the wall. The effective capacitance C_{eff} is given by

$$\frac{1}{C_{eff}} = \int ds \, \frac{1}{C_{sheath}(s)} + \frac{1}{C_{plasma}} \tag{A1}$$

where,

$$\frac{1}{C_{plasma}} = \frac{1}{C_o\left(1 - \dfrac{\omega_p^2}{\omega^2 - \omega_c^2}\right)} \tag{A2}$$

and

$$\frac{1}{C_{sheath}(s)} = \frac{1}{C_o\left(1 - \frac{\omega_p^2(s)}{\omega^2 - \omega_c^2}\right)} . \tag{A3}$$

The gyrofrequency ω_c is independent of electron density. The plasma frequency $\omega_p(s)$ is given by

$$\omega_p^2(s) = \frac{4\pi e^2 N(s)}{m} , \tag{A4}$$

where a general realistic model of N(s) is shown in Figure A1.

$$N(s) = N_o (1 - \exp(-\alpha s)) , \tag{A5}$$

where α is a sheath parameter of electron density profile.

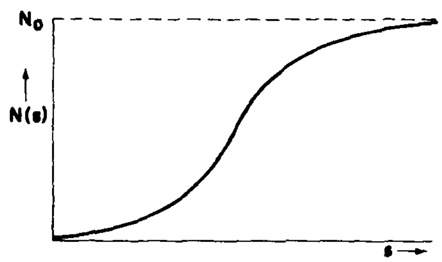

Figure A1. Sheath Electron Density Profile

Consider the integral

$$I = \int_0^d ds \frac{1}{a - b \exp(-\alpha s)} \tag{A6}$$

where $a > 0$, $b > 0$, and $\alpha > 0$. Let $r = \exp(-\alpha s)$. Integrating over r. One obtains

$$I = \frac{1}{a\alpha}\Big[\ln |a - br| - \ln r\Big]_{s=0}^{s=d} . \tag{A7}$$

Substituting $a = C_o$ and $b = C_o N_o d^2/m(\omega^2 - \omega_c^2)$ into Eq. (A7) and using Eqs. (A1), (A2), and (A3), one obtains

$$\frac{1}{C_{eff}} = \frac{1}{C_o}\left\{\frac{\omega^2 - \omega_c^2}{\omega^2 - (\omega_c^2 + \omega_p^2)} + \frac{2}{\alpha}\ln\left|\frac{\omega^2 - \omega_c^2 - \omega_p^2 \exp(-\alpha d)}{\omega^2 - (\omega_c^2 + \omega_p^2)}\right| + 2\alpha d\right\}, \tag{A8}$$

where the factor 2 is due to the two sheaths in a parallel plate condenser (Figure 3).

The admittance is obtained by using Eq. (A8):

$$\frac{1}{Z(\omega)} = j\omega C_o\left\{\frac{\omega^2 - \omega_{UHF}^2}{\omega^2 - \omega_c^2 + (\omega^2 - \omega_{UHF}^2) F(\alpha)}\right\} , \tag{A9}$$

where

$$F(\alpha) = \frac{2}{\alpha}\ln\left|\frac{\omega^2 - \omega_c^2 - \omega_p^2 \exp(-\alpha d)}{\omega^2 - \omega_{UHF}^2}\right| + 2\alpha d . \tag{A10}$$

From Eq. (A9) it is readily observed that the zero of admittance is still at ω_{UHF} while the pole is a function of sheath parameter α.

In the limiting case that α is small so that

$$\lim_{\alpha \to 0} F(\alpha) = 0 . \tag{A11}$$

The pole of $1/Z(\omega)$ becomes at ω_c, agreeing with the limiting result in Eq. (10) with no sheath [$s \to 0$ in Eq. (10)]. The conjecture is proved.

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